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SOLUTION OF PROBLEM 118.

BY PROF. JOHNSON.

DENOTING the pairs of opposite sides and diagonals by a, a' ; b, b' ; and c, c' ; and the angles between b and c , c and a , and a and b respectively by α, β and γ we have $\alpha + \beta = \gamma$, from which we derive the relation between cosines

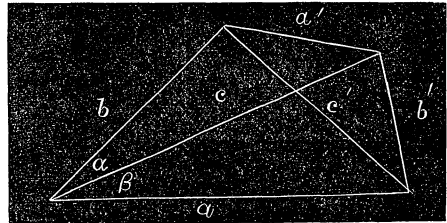
$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma - 2 \cos \alpha \cos \beta \cos \gamma = 1. \dots (1)$$

By the triangles

$$\cos \alpha = \frac{b^2 + c^2 - a'^2}{2bc},$$

$$\cos \beta = \frac{c^2 + a^2 - b'^2}{2ac},$$

$$\cos \gamma = \frac{a^2 + b^2 - c'^2}{2ab}.$$



Substituting in (1)

$$b^2(c^2 + a^2 - b'^2)^2 + a^2(b^2 + c^2 - a'^2)^2 + c^2(a^2 + b^2 - c'^2)^2 - (c^2 + a^2 - b'^2)(b^2 + c^2 - a'^2) \times (a^2 + b^2 - c'^2) - 4a^2b^2c^2 = 0,$$

which reduces to

$$a^2a'^2(a^2 + a'^2 - b^2 - b'^2 - c^2 - c'^2) + b^2b'^2(b^2 + b'^2 - c^2 - c'^2 - a^2 - a'^2) + c^2c'^2(c^2 + c'^2 - a^2 - a'^2 - b^2 - b'^2) + a^2b^2c'^2 + b^2c^2a'^2 + c^2a^2b'^2 + a'^2b'^2c'^2 = 0. \dots (2)$$

In the special case where $a = a'$ and $b = b'$ (2) reduces to

$$2a^4(a^2 - b^2) - 2b^4(a^2 - b^2) - (a^4 + b^4 - 2a^2b^2 + c^2c'^2)(c^2 + c'^2) - 2(a^2 + b^2)c^2c'^2 = 0$$

$$\text{or} \quad [c^2c'^2 - (a^2 - b^2)^2][c^2 + c'^2 - 2(a^2 + b^2)] = 0.$$

$$\text{Hence, either} \quad c^2 + c'^2 = 2(a^2 + b^2), \dots (3)$$

$$\text{or} \quad cc' = a^2 - b^2. \dots (4)$$

(3) holds when a, a', b and b' form a parallelogram and (4) holds when these lines form the crossed quadrilateral known as the contra-parallelogram, c and c' are parallel and have a constant product, as first observed by Mr. Henry Hart, who employed a linkage of this form as a reciprocator. (See ANALYST, Vol. III, p. 46 or Vol. II, p. 44.)

[As Prof. Johnson claims that the solution of this question, as published at page 126, is not sufficiently general to meet the requirements of the question as proposed, we insert the above and add the remark, that the conclusion of Prof. Scheffer's solution, in which he had discussed the crossed quadrilateral, was omitted in the printed solution to avoid the necessity of introducing another diagram.]